On con ergence of dyn cs of hopp ng p rt cl es to rth nd de th process n cont n

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Abstract

We show that some classes of birth-and-death processes in continuum

Here and below, for simplicity of notations, we just write $\mathbf{x}, \mathbf{y}$ instead of $\{\mathbf{x}\}$

Let us briefly recall some basic facts of harmonic analysis on the configuration space, see [,] for further details. Let Γ_0 denote the space of all finite configurations in $\mathbb{R}^d$, i.e., $\Gamma_0 := \bigsqcup_{n=0}^{\infty} \Gamma^{(n)}$, where $\Gamma^{(n)}$ is the space of all n-point configurations in $\mathbb{R}^d$. Clearly, $\Gamma_0 \subset \Gamma$, and we define $\mathcal{B}(\Gamma_0)$ and $\mathcal{B}(\Gamma^{(n)})$ as the trace σ-algebra of $\mathcal{B}(\Gamma)$ on Γ_0 and $\Gamma^{(n)}$, respectively. For a function $G:\Gamma_0\to\mathbb{R}$, we define a function $KG:\Gamma\to\mathbb{R}$ by $(KG)(\gamma) := \sum_{\eta\Subset\gamma} G(\eta)$, $\gamma\in\Gamma$, provided the summation makes sense. Here $\eta\Subset\gamma$ means that η is a finite subset of γ.

Let μ be a probability measure on $(\Gamma,\mathcal{B}(\Gamma))$. Then there exists a unique measure ρ on $(\Gamma_0,\mathcal{B}(\Gamma_0))$ satisfying

$$\int_{\Gamma} (KG)(\gamma)\,\mu(d\gamma) = \int_{\Gamma_0} G(\eta)\,\rho(d\eta)$$

for each measurable function $G:\Gamma_0\to[0,\infty]$. The measure ρ is called the correlation measure of μ. Further, denote by λ the Lebesgue–Poisson measure on Γ_0, i.e.,

$$\lambda := \delta_{\varnothing} + \sum_{n=1}^{\infty} \frac{1}{n!}\, dx_1\cdots dx_n.$$

Here $\delta_{\varnothing}$ is the Dirac measure with mass at $\varnothing$, and $dx_1\cdots dx_n$ is the Lebesgue measure on $\Gamma^{(n)}$, which is naturally defined on this space. Assume that the correlation measure ρ of μ is absolutely continuous with respect to λ. Then $k := \frac{d\rho}{d\lambda}$ is called the correlation functional of μ. For a given correlation functional k, the corresponding Ursell functional $u:\Gamma_0\to\mathbb{R}$ is defined through the formula $k(\eta) = \sum_{\pi\in\mathcal{P}(\eta)} u_\pi(\eta)$, where $\mathcal{P}(\eta)$ denotes the set of all partitions of η, and given a partition $\pi = \{\eta_1,\dots,\eta_k\}$ of η, $u_\pi(\eta) := u(\eta_1)\cdots u(\eta_k)$. Recall also that a function $G:\Gamma_0\to\mathbb{R}$ is called translation invariant if, for each $x\in\mathbb{R}^d$, $G(\eta_x) = G(\eta)$ for all $\eta\in\Gamma_0$, where η_x denotes the configuration η shifted by vector x, i.e., $\eta_x := \{y+x \mid y\in\eta\}$. Clearly, the correlation functional k is translation invariant if and only if the corresponding Ursell functional u is translation invariant.

If k is the correlation functional of a probability measure μ on Γ, we denote

$$k^{(n)}(x_1,\dots,x_n) := k(\{x_1,\dots,x_n\}), \quad n\in\mathbb{N},$$

and analogously we define $u^{(n)}$. The $(k^{(n)})_{n=1}^{\infty}$ and $(u^{(n)})_{n=1}^{\infty}$ are called the correlation and Ursell functions of μ, respectively. Note that, if k is translation invariant, then $k^{(1)} = u^{(1)}$ is a constant.

For a function $f:\mathbb{R}^d\to\mathbb{R}$, we define $e_\lambda(f,\eta) := \prod_{x\in\eta} f(x)$, $\eta\in\Gamma_0$

Assume that $\mathbf{L}$ is a Markov generator on Denote $\mathbf{L} \cdot \mathbf{K}^{-1}\mathbf{L}\mathbf{K}$, i.e., $\mathbf{L}$ is the operator acting on functions on $_0$ which satisfies $\mathbf{K}\mathbf{L}\mathbf{G}$ $\mathbf{L}\mathbf{K}\mathbf{G}$ Denote by $\mathbf{L}$

Proof. A straightforward calculation (see) shows that

$$
\begin{aligned}
L_\varepsilon^{-*}k(\eta) &= -\sum_{x\in\eta}\int_{\mathbb{R}^d} dy\, a_\varepsilon(x-y)\, r(x,y,\eta\setminus x)\\
&\quad\times\int_{\Gamma_0}\lambda(d\xi)\, k(\xi\cup\eta)\, e_\lambda\big(e^{\varphi^-(x-\cdot)-\varphi^+(y-\cdot)}-1,\xi\big),\\
L_\varepsilon^{+*}k(\eta) &= \sum_{y\in\eta}\int_{\mathbb{R}^d} dx\, a_\varepsilon(x-y)\, r(x,y,\eta\setminus y)\\
&\quad\times\int_{\Gamma_0}\lambda(d\xi)\, k(\xi\cup(\eta\setminus y)\cup x)\, e_\lambda\big(e^{\varphi^-(x-\cdot)-\varphi^+(y-\cdot)}-1,\xi\big),\\
L_0^{-*}k(\eta) &= -\sum_{x\in\eta}\exp\big(E^{\varphi^-}(x,\eta\setminus x)\big)\\
&\quad\times\int_{\Gamma_0}\lambda(d\xi)\, e_\lambda\big(e^{\varphi^-(x-\cdot)}-1,\xi\big)\, k(\eta\cup\xi),\\
L_0^{+*}k(\eta) &= \sum_{y\in\eta}\exp\big(-E
\end{aligned}
$$

Fourier transforms so that they are unitary operators in L $\mathbb{R}^d \to \mathbb{C}, dx$. For any $\gamma \in$, consider a dynamics of independent particles which starts at γ and such that each separate particle moves according to the semigroup p_t, i.e., independent random walks in $\mathbb{R}^d$. Then, this process has *cádlág* paths on and

and initial distribution μ_0. This dynamics can be constructed as follows, cf. [ ]. For each $\gamma \in$, denote by P_γ the law of a process on which is at γ at time zero, and after this, points of γ randomly die, independently of each other, so that the probability that at time $t >$ a particle $x \in \gamma$ is still alive is equal to e^{-t}. Next, let π denote the Poisson point process in $\mathbb{R}^d \times [\ ,\infty)$ with the intensity measure $k_0^{(1)}\,dx\,dt$. The measure π is concentrated on configurations $\hat\gamma = \{(x_n, t_n)\}_{n=1}^\infty$ in $\mathbb{R}^d \times [\ ,\infty)$ such that $\{x_n\}_{n=1}^\infty \in$, $< t_1 < t < \cdots$, and $t_n \to \infty$ as $n \to \infty$. For any such configuration, we denote by $P_{\hat\gamma}$ the law of a process on such that at time t , the configuration is empty, and then at each time t_n, $n \in \mathbb{N}$, a new particle is born at x_n, and after time t_n this particle randomly dies, independently of the other particles, so that at time $s > t_n$ the probability that the particle is still alive is $e^{-(s-t_n)}$. Finally, the law of the process with generator and initial distribution μ_0 is given by

$$\int \mu_0(d\gamma)\, P_\gamma * \int \pi(d\hat\gamma)\, P_{\hat\gamma}.$$

Here $*$ stays for convolution of measures, see [ ] for details.

gro p sca led by ε Set

$$g^{\varepsilon}(x) = e^{f_0(x)} \int_{\mathbb{R}^d} p^{\varepsilon}_{t_1}(x, dx_1) \int_{\mathbb{R}^d} p^{\varepsilon}_{t_2-t_1}(x_1, dx_2) \times \cdots \times \int_{\mathbb{R}^d} p^{\varepsilon}_{t_n-t_{n-1}}(x_{n-1}, dx_n) \prod_{i=1}^{n} e^{f_i(x_n)}, \quad x \in \mathbb{R}^d.$$

hen, by and the constr ction of the process, the first integral in ith $\varepsilon >$ is eq al to

$$\int \prod_{x \in \gamma} g^{\varepsilon}(x)\, \mu_0(d\gamma) = \sum_{n=0}^{\infty} \frac{1}{n!} \int_{(\mathbb{R}^d)^n} \prod_{i=1}^{n} (g^{\varepsilon}(x_i) - 1)\, k_0^{(n)}(x_1, \dots, x_n)\, dx_1 \cdots dx_n.$$

n the abo e integrals, one represents the correlation f nctions thro gh the rsell f nctions, a es a change of aria les nder the sign of integral, and after a caref l analysis of the obtained e pression, one ta es its li it as $\varepsilon \to$ Finally, one sho s that the obtained li it is indeed eq al to the second integral in

Convergence of equi i riu w s ki dyn ics of inter cting p rtic es

n this section, e ill consider eq ili ri dyna ics of interacting particles ha ing a Gi bs eas re as an eq ili ri eas re O r res lt ill e tend that of , here first one special case of s ch a dyna ics as considered see also e start ith a description of the class of Gi bs eas res e are going to se

A pair potential is a Borel eas ra le f nction $\varphi : \mathbb{R}^d \to \mathbb{R} \cup \{+\infty\}$ s ch that $\varphi(-x) = \varphi(x) \in \mathbb{R}$ for all $x \in \mathbb{R}^d \setminus \{0\}$ For $\gamma \in$ and $x \in \mathbb{R}^d \setminus \gamma$, e define a relati e energy of interaction bet een a particle at x and the config ration γ as $E(x, \gamma) := \sum_{y \in \gamma} \varphi(x - y)$, pro ided that the latter s con erges absol tely, and other ise it is set to be ∞ A grand canonical Gi bs eas re corresponding to the pair potential φ and acti ity $z >$

n particular, e then ha e $\varphi\ x \ \geq - B$, $x \in \mathbb{R}^d$ Ne t, e say that the condition of lo acti ity high te perat re regi e is f lfilled if

$$\int_{\mathbb{R}^d} |e^{-\varphi\ x)} - \ |z\, dx < \ e^{1+\ B\ -1},$$

here B is as in A classical res lt of R elle , says that, nder the ass ption of stability and lo acti ity high te perat re regi e, there e ists a Gibbs eas re μ corresponding to φ and z, and this eas re has correlation f nctional hich satisfies conditions i iii of heore , ith s in condition i hich is then called the R elle bo nd F rther ore, the corresponding rsell f nctions satisfy $u^{\ n)} \ ,\cdot,\ldots,\cdot \in L^1 \ \mathbb{R}^{d\ n-1}, dx_1 \cdots dx_n$ for each $n \geq$ n hat follo s, e ill ass e that the potential φ is also bo nded fro abo e o tside so e finite ball in $\mathbb{R}^d$ hich is al ays tr e for any realistic potential, since it sho ld con erge to zero at infinity p§ths hose gener§tor is $L_K, D\ L_K$ f e consider this process ith initi§l distri tion μ, then it is §n eq i ri process, i e , it h§s distri tion μ_t μ §t §ny o ent of ti e $t \geq$ h s, for e§ch $t \geq$, μ_t μ h§s corre §tion f nction hich s§tis es conditions i iii of heore

respect to μ, as well as integrals over $\mathbb{R}^d$ with respect to Lebesgue measure. As a result one gets rid of all s tions $_{x,y}$ hen, one es a change of

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